COT 3100 In-class Exercise 4

Name: USF ID:

Problem 1 Prove the following statements by direct proof.

1. The product of any two odd integers is odd.

Proof:

Suppose that and are any particular and arbitrarily chosen odd numbers. [We must show that is odd].

By definition of odd, and for some integersand. Then,

by algebra.

Let and is an integer because products and sums of integers are integers and 2,, and are integers. Hence, where is an integer, and so, by definition of odd, is odd.

1. For any integers and, if is even, then is even.

Proof:

Suppose that and are any particular and arbitrarily chosen integers such that is even. [We must show is even].

By definition of even, for some integer. Then,

Let, and is an integer because products and sums of integers are integers and,, and are integers. Hence, where is an integer, and so, by definition of even, is even.

1. For all integers, if, then is composite.

Proof:

Suppose that is any particular and arbitrarily chosen integer such that. [We must show is composite].

by substitution

Let and. and are integers because sums of integers are integers and 2 and are integers. Hence,, where and are integers. and, and since. Also, and since, and. Hence, and. So, is composite by definition of composite.

1. The square of any rational number is rational.

Proof:

Suppose that is any particular and arbitrarily chosen rational number. By definition of rational number, for some integers and, with.

By substitution,

.

Since and are both integers, so are the products and. Also by the zero product property. Hence, is a ratio of two integers with a nonzero denominator, and so is rational by definition of rational.